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On the non-unique definition of the current in the Thirring model

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Abstract. It is shown that in the Thirring model there exists a more general definition of the current than the usual one, which leads to a one-parameter family of renormalised solutions of the model. The set of vector currents obtained in this way yields many possible quantisations of the model, and in particular a quantisation with a finite renormalisation. It is also shown that with such a definition of the current the conformal dimension of the two-point function is not fixed.

1. Introduction

In a number of papers (Mandelstam 1975, Pogrebkov and Sushko 1975, 1976, Hadjiivanov *et al* 1979) an exact solution of the renormalised massless quantum Thirring model has been considered in the form of an exponential of two massless scalar fields. Usually in the literature one finds the exact operator solution of Klaiber (1967), although its conformal properties have not been treated. It is known that this solution is not unique. In the paper by Kupsch *et al* (1975), for instance, a solution different from Klaiber's has been obtained, and the authors express the opinion that a full set of solutions is necessary to make the conformal invariance properties of the Thirring model clear. Independently, another solution has been obtained by Dell'Antonio *et al* (1972), and in the works by Streater and Wilde (1970) and Streater (1974), a two-dimensional model is studied which has such a solution in the proper limit. As has been shown by Hadjiivanov *et al* (1979), the Thirring model is invariant with respect to transformations of the conformal group which are not the standard ones. Therefore its solutions are not spinors (nor what are usually called spinors in two-dimensional space–time).

Here it is necessary to stress that in two-dimensional space–time there exist no spinors at all, because the Lorentz group is abelian and all its irreducible representations are one-dimensional. As a consequence of that fact, the usual spin-statistics connection is not valid (see Streater and Wilde (1970), Streater (1974)). Therefore objects exist which transform with respect to the Lorentz group as $\psi_i(x) \rightarrow [(\exp A\chi\gamma_5)\psi]_i(x)$, where A is an arbitrary constant. It is clear that the value $A = \frac{1}{2}$ has been chosen only in analogy to the four-dimensional case. Even the analogue of the free quantised Dirac equation in two-dimensional space–time

$$i\gamma^{\mu}\partial_{\mu}\psi(x) = 0 \qquad (1.1)$$

has solutions for which the constant A can take arbitrary values (see Kupsch *et al* (1975)).

Remark. The value $A=\frac{1}{2}$ is fixed by the requirement for canonical commutation relations between the fields $\psi(x)$. However, as is known, the solutions of the Thirring model do not satisfy such commutation relations. As is seen from the works of Mandelstam (1975), Pogrebkov and Suchko (1975, 1976) and Hadjiivanov *et al* (1979), the canonical commutators can only be given for the scalar field operators $\phi(x)$ (with the help of which $\psi(x)$ is expressed). For a consistent study of the Thirring model and the corresponding free 'spinor' field, it is also necessary in the latter case to require canonical commutation relations between the fields $\phi(x)$, and not between $\psi(x)$. Then all the solutions of equation (1.1) corresponding to different values of the constant A are equally valid.

As is known, the solution of the Thirring model is connected with two conserved currents (a vector and pseudovector current) which are expressed as gradients of the two scalar fields. In a paper by Johnson (1961), a correct definition of the vector current has been given corresponding to the particular value of the constant $A=\frac{1}{2}$. In the present paper we find an analogous definition of the current for the case of arbitrary values of A. It will be seen that for some values of A the Thirring model needs no infinite renormalisation.

We note further that Klaiber's solution (which is not restricted by the fixed value $A=\frac{1}{2}$ either) is constructed in the Hilbert space of the free spinor field, and a substitute for the scalar field is defined through a regularisation procedure. The vacuum in this space is unique, and it is invariant with respect to the gauge transformations generated by the vector and the pseudovector currents of the Thirring model. In particular, the vacuum expectation value of the Thirring field is equal to zero, and the matrix structure of the two-point function is determined by the matrix structure of the free spinor field two-point function. The solutions proposed here are constructed only in the space of the scalar field operator states where the Thirring field operator $\psi(x)$ is expressed in the form of an exponential of these fields. In the conformal invariant, consistent field theory of the Thirring model which arises with the help of such a construction, a spontaneous breakdown of the gauge γ_5-symmetry takes place. We will discuss this problem in detail in § 3.

2. The current in the Thirring model

Let us first write the exact solution of the renormalised massless quantum Thirring model obtained by Hadjiivanov *et al* (1979) as an exponential of two massless scalar fields:

$$\begin{aligned}\psi_k(x) &= [\exp(\mathrm{i}\beta\gamma_5\tilde{\phi}^-(x)-\mathrm{i}\alpha\phi^-(x))\exp(-\mathrm{i}\alpha\phi^+(x)+\mathrm{i}\beta\gamma_5\tilde{\phi}^+(x))u]_k\\ &= \exp(\mathrm{i}\beta(-1)^k\tilde{\phi}^-(x)-\mathrm{i}\alpha\phi^-(x))\exp(-\mathrm{i}\alpha\phi^+(x)+\mathrm{i}\beta(-1)^k\tilde{\phi}^+(x))u_k \qquad (2.1)\end{aligned}$$

where $|u_1|^2=|u_2|^2=|u|^2$ and

$$\Box\phi(x)=0, \qquad \gamma^0=\sigma_1, \qquad \gamma^1=\mathrm{i}\sigma_2, \qquad \gamma^5=\gamma^0\gamma^1. \qquad (2.2)$$

σ_1 and σ_2 are the Pauli matrices. We also write the non-zero commutators of the field

$\phi(x)$ and its positive- and negative-frequency parts $\phi^{\pm}(x)$:

$$[\phi(x), \phi(y)] = \mathrm{i}D(x-y), \tag{2.3}$$

$$[\phi^{\pm}(x), \phi^{\mp}(y)] = D^{\pm}(x-y). \tag{2.4}$$

The dual field $\tilde{\phi}(x)$ (pseudoscalar) also satisfies equation (2.2). Both fields are connected through a linear differential equation:

$$\partial_\mu \phi(x) + \epsilon_{\mu\nu}\partial^\nu \tilde{\phi}(x) = 0, \qquad \epsilon_{\mu\nu} = -\epsilon_{\nu\mu}, \qquad \epsilon_{01} = -\epsilon^{01} = 1. \tag{2.5}$$

The field $\tilde{\phi}(x)$ and its positive- and negative-frequency parts satisfy the same commutation relations (2.3) and (2.4) and commute with the fields $\phi(x)$ and $\phi^{\pm}(x)$ in the following way:

$$[\tilde{\phi}(x), \phi(y)] = \mathrm{i}\tilde{D}(x-y), \tag{2.6}$$

$$[\tilde{\phi}^{\pm}(x), \phi^{\mp}(y)] = \tilde{D}^{\pm}(x-y), \qquad [\tilde{\phi}^{\pm}(x), \phi^{\pm}(y)] = 0. \tag{2.7}$$

The commutator functions are given in appendix 1. Usually the current in the Thirring model has been defined following Johnson (1961). We recall this definition. First of all one considers the expressions

$$j_\mu(x) = \lim_{\substack{\epsilon^0=0\\ \epsilon^1\to 0}} j_\mu(x, \epsilon), \qquad \tilde{j}_\mu(x) = \lim_{\substack{\epsilon^0=0\\ \epsilon^1\to 0}} j_\mu(x, \tilde{\epsilon}) \tag{2.8}$$

where

$$\begin{aligned} j_\mu(x, \epsilon) &= (-\epsilon^2)^{(1/4\pi)(\alpha^2+\beta^2)-1/2}[\bar{\psi}(x+\epsilon)\gamma_\mu\psi(x) - \psi(x)\gamma_\mu^{\mathrm{T}}\bar{\psi}(x-\epsilon)], \\ \tilde{\epsilon}^2 &= -\epsilon^2, \qquad \tilde{\epsilon}\epsilon = 0 \end{aligned} \tag{2.9}$$

(we are discussing here the renormalised Thirring model). Then one obtains the current in the form

$$J_\mu(x) = \tfrac{1}{2}[j_\mu(x) + \tilde{j}_\mu(x)]. \tag{2.10}$$

The renormalised Thirring model with such a current leads to solutions of the type (2.1), with α and β satisfying

$$\alpha\beta = \pi, \qquad \beta - \alpha = (g/2\pi)(\alpha+\beta). \tag{2.11}$$

The current is expressed with the help of the field $\phi(x)$ as

$$J_\mu(x) = (1/2\pi)(\alpha+\beta)\partial_\mu\phi(x). \tag{2.12}$$

From equations (2.11) the constants α and β are uniquely determined.

On the other hand it is not difficult to calculate the expressions (2.8) without fixing α and β. Let us write the components of the current explicitly:

$$\begin{aligned} j_0(x) &= -(\mathrm{i}/2\pi)(-1)^{\alpha\beta/2\pi}\{(-1)^{-\alpha\beta/\pi}[-\alpha\partial_1\phi - \beta\partial_1\tilde{\phi}] - \alpha\partial_1\phi + \beta\partial_1\tilde{\phi}\}, \\ j_1(x) &= -(\mathrm{i}/2\pi)(-1)^{\alpha\beta/2\pi}\{(-1)^{-\alpha\beta/2\pi}[-\alpha\partial_1\phi - \beta\partial_1\tilde{\phi}] + \alpha\partial_1\phi - \beta\partial_1\tilde{\phi}\}, \end{aligned} \tag{2.13}$$

$$\tilde{j}_0(x) = (1/\pi)\alpha\partial_1\tilde{\phi}, \qquad \tilde{j}_1(x) = (1/\pi)\beta\partial_1\phi, \tag{2.14}$$

where we have set $(-1)^a = \mathrm{e}^{\mathrm{i}\pi a}$ and $(\mu^2)^{-(\alpha^2+\beta^2)/4\pi}|u|^2 = 1/2\pi$.

It is readily seen that equations (2.13) and (2.14) give a one-to-one correspondence between $j_\mu(x)$ and $\tilde{j}_\mu(x)$ on the one hand and $\alpha\partial_1\phi$, $\beta\partial_1\phi$ and $\alpha\partial_1\tilde{\phi}$, $\beta\partial_1\tilde{\phi}$ on the other. Therefore, without fixing α and β, one can solve equations (2.13) and (2.14) with respect to $\partial_\mu\phi(x)$ and $\partial_\mu\tilde{\phi}(x)$ and construct the current with the help of equation (2.12), considering the latter equation as its definition. We write the result explicitly:

$$\begin{aligned}(1/2\pi)(\alpha+\beta)\partial_0\phi &\equiv J_0(x)\\ &=\tfrac{1}{2}\{\tilde{j}_0(x)+\tfrac{1}{2}\mathrm{i}(-1)^{-\alpha\beta/2\pi}[j_0(x)-j_1(x)]-\tfrac{1}{2}\mathrm{i}(-1)^{\alpha\beta/2\pi}[j_0(x)+j_1(x)]\},\end{aligned} \tag{2.15}$$

$$\begin{aligned}(1/2\pi)(\alpha+\beta)\partial_1\phi &\equiv J_1(x)\\ &=\tfrac{1}{2}\{\tilde{j}_1(x)-\tfrac{1}{2}\mathrm{i}(-1)^{-\alpha\beta/2\pi}[j_0(x)-j_1(x)]-\tfrac{1}{2}\mathrm{i}(-1)^{\alpha\beta/2\pi}[j_0(x)+j_1(x)]\}.\end{aligned} \tag{2.16}$$

Thus, if the current of the Thirring model is determined with the help of equations (2.15) and (2.16), there remains one relation between the constants α and β:

$$\beta-\alpha=(g/2\pi)(\alpha+\beta). \tag{2.17}$$

The definitions (2.15) and (2.16) differ from the Johnson one and coincide with it if $\alpha\beta=\pi$. Equation (2.17) does not determine the constants α and β uniquely, and therefore equation (2.1) represents a one-parameter family of renormalised solutions of the Thirring model. As is known, the factor $(-\epsilon^2)^{-(\alpha^2+\beta^2)/8\pi+1/4}(\epsilon\to 0)$ has the meaning of a renormalisation constant for the field operators. More precisely, the renormalised fields $\psi(x)$ and the unrenormalised ones $\psi_0(x)$ are related through

$$\psi(x)\sim(-\epsilon^2)^{-(1/8\pi)(\alpha^2+\beta^2)+1/4}\psi_0(x). \tag{2.18}$$

As far as there exists only one relation (2.17) between α and β, then equation (2.18) provides the Thirring model with a one-parameter family of renormalisations. For instance, the constant α can be chosen as a parameter.

Any solution belonging to the family for a given α can be expressed with the help of the scalar fields $\phi(x)$ and $\tilde{\phi}(x)$ according to equation (2.1), and therefore for any α the Thirring equation is invariant with respect to the representations of the conformal group with generators given in appendix 2. The Thirring fields $\psi(x,\alpha)$ have no fixed spin, and therefore the spin-statistics relation is not uniquely determined. At space-like separations commutation or anticommutation relations hold if the spin is integer or half-integer respectively. The problem of local commutativity of these solutions is discussed by Hadjiivanov and Stoyanov (1979).

We point out that the two-point function of the fields $\psi(x)$ has no fixed conformal dimension, because it depends on α and hence on the chosen renormalisation. Changing the value of the parameter α, one goes from one solution to another, i.e. roughly speaking from one dimension to another. For instance, the transformation

$$\psi(x,\alpha)\to :\exp\{\mathrm{i}(\varkappa-1)[\beta\gamma_5\tilde{\phi}(x)-\alpha\phi(x)]\}\psi(x\;\alpha): \tag{2.19}$$

reduces to a multiplication of α and β by a factor $\varkappa$,

$$\psi(x,\alpha)\to\psi(x,\varkappa\alpha), \tag{2.20}$$

which does not alter equation (2.17). Therefore the whole family of renormalised Thirring equations remains invariant with respect to the transformation (2.19).

We show now that the expressions for the current components (2.15) and (2.16) can be written in a manifestly covariant form. For this purpose we introduce the quantities

$$T_\mu(x,\epsilon)=\tfrac{1}{2}[V_1(\epsilon)(\delta_\mu{}^\nu-\epsilon_\mu{}^\nu)+V_2(\epsilon)(\delta_\mu{}^\nu+\epsilon_\mu{}^\nu)]j_\nu(x,\epsilon) \tag{2.21}$$

where $j_\nu(x, \epsilon)$ is given by equation (2.9) (α and β are arbitrary), and $V_1(\epsilon)$ and $V_2(\epsilon)$ are given by

$$V_1(\epsilon)=\left(\frac{\epsilon^0-\epsilon^1-\mathrm{i}0}{\epsilon^0+\epsilon^1-\mathrm{i}0}\right)^{-\alpha\beta/2\pi}=[V_2(\epsilon)]^{-1}. \tag{2.22}$$

It is easily verified that

$$\lim_{\substack{\epsilon^0=0\\ \epsilon^1\to 0}} T_\mu(x,\epsilon)=\tfrac{1}{2}\{(-1)^{-\alpha\beta/2\pi}[j_0(x)+j_1(x)]+(-1)^{\alpha\beta/2\pi+\mu}[j_0(x)-j_1(x)]\}\equiv T_\mu(x),$$

$$\lim_{\substack{\epsilon^0=0\\ \epsilon^1\to 0}} T_\mu(x,\tilde{\epsilon})=\tilde{j}_\mu(x)\equiv \tilde{T}_\mu(x).$$

Therefore the current (2.15) and (2.16) can be written in a covariant form:

$$J_\mu(x)=\tfrac{1}{2}[\tilde{T}_\mu(x)+\mathrm{i}\epsilon_\mu{}^\nu T_\nu(x)]. \tag{2.23}$$

3. Conformal invariant two-point functions

In this section we discuss the conformal invariant two-point functions of the fields $\psi(x)$. Consider the fields $\psi^1(x)$ and $\psi^2(x)$ (with parameters α_1, β_1 and α_2, β_2 respectively), which transform according to the representation of the conformal group given in appendix 2. The two-point function is

$$\Delta_{ij}(x)=\langle 0|\psi_i^1(x)\bar{\psi}_j^2(0)|0\rangle \tag{3.1}$$

($\bar{\psi}$ is the Dirac conjugate field). The Lorentz invariance condition equation (A2.2) leads to the equation

$$(M_{\mu\nu}\Delta)_{ij}\equiv -\mathrm{i}(x_\mu\partial_\nu-x_\nu\partial_\mu)\Delta_{ij}(x)+(\mathrm{i}/2\pi)\epsilon_{\mu\nu}\alpha_2\beta_1(\gamma_5\Delta)_{ij}-(\mathrm{i}/2\pi)\epsilon_{\mu\nu}\alpha_1\beta_2(\Delta\gamma_5)_{ij}=0. \tag{3.2}$$

Analogously, using equation (A2.3), one obtains the scale invariance condition of the two-point function

$$(D\Delta)_{ij}(x)\equiv \mathrm{i}x^\mu\partial_\mu\Delta_{ij}(x)+(\mathrm{i}/2\pi)\alpha_1\alpha_2\Delta_{ij}(x)-(\mathrm{i}/2\pi)\beta_1\beta_2(\gamma_5\Delta\gamma_5)_{ij}=0. \tag{3.3}$$

Further, equation (A2.4) gives the invariance condition with respect to the special conformal transformations $(K_\mu\Delta)_{ij}=0$. However, this condition does not lead to new equations for the function $\Delta_{ij}(x)$, but is rather an identity:

$$(K_\mu\Delta)_{ij}(x)=x^\nu(M_{\mu\nu}\Delta)_{ij}(x)-x_\mu(D\Delta)_{ij}(x). \tag{3.4}$$

It is not difficult to solve the system of equations (3.2) and (3.3). One makes the substitution

$$u=x^+x^-,\qquad v=x^-/x^+,\qquad \Delta_{ij}(x)=\Delta_{ij}(u,v), \tag{3.5}$$

where $x^+=x^0-\mathrm{i}0+x^1$, $x^-=x^0-\mathrm{i}0-x^1$. Then equations (3.2) and (3.3) take a very simple form:

$$v\frac{\partial}{\partial v}\Delta_{ij}(u,v)=\frac{1}{4\pi}[\alpha_1\beta_2(-1)^j-\alpha_2\beta_1(-1)^i]\Delta_{ij}(x), \tag{3.6}$$

$$u\frac{\partial}{\partial u}\Delta_{ij}(u,v)=\frac{1}{4\pi}[\beta_1\beta_2(-1)^{i+j}-\alpha_1\alpha_2]\Delta_{ij}(x). \tag{3.7}$$

In obtaining the last equations we made use of the concrete realisation of the γ_5-matrix, namely $(\gamma_5)_{kl}=(-1)^k\delta_{kl}$. The final result is

$$\Delta_{ij}(x)=H_{ij}(-\mu^2x^2+\mathrm{i}0x^0)^{(1/4\pi)[\beta_1\beta_2(-1)^{i+j}-\alpha_1\alpha_2]}\times\left(\frac{x^0-x^1-\mathrm{i}0}{x^0+x^1-\mathrm{i}0}\right)^{(1/4\pi)[\alpha_1\beta_2(-1)^j-\alpha_2\beta_1(-1)^i]} \tag{3.8}$$

where H_{ij} are some integration constants. The functions (3.8) are different from zero for any α_1, β_1 and α_2, β_2. Thus the two-point functions of the Thirring fields are different from zero for different renormalisation constants. Note that functions of the kind

$$\tilde{\Delta}_{ij}(x)=\langle 0|\psi_i^1(x)\psi_j^2(0)|0\rangle \tag{3.9}$$

do not vanish either. The explicit expression for the latter is obtained from (3.2) after substituting $\alpha_2\to-\alpha_2$. The fact that there exists a two-point function of the fields $\psi_i^1(x)$ and $\bar{\psi}_j^2(x)$ actually means that for fixed α and β the algebra (A2.1)–(A2.4) determines a series of irreducible representations due to the continuous spectrum of the operators L and S. The sets of irreducible representations contained in two series (with different α and β) evidently coincide, which reflects in the non-vanishing two-point function of the fields $\psi_i^1(x)$ and $\psi_j^2(0)$.

Obviously $\Delta_{ij}(x)$ are homogeneous functions of x, but the degree of homogeneity depends on the value of the indices i and j. Namely, the diagonal terms of the matrix $\Delta_{ij}(x)$ have one degree of homogeneity $-(1/2\pi)(\beta_1\beta_2-\alpha_1\alpha_2)$, and the antidiagonal have another one, $-(1/2\pi)(\beta_1\beta_2+\alpha_1\alpha_2)$. Nevertheless the two-point function of the field $\psi(x)$ is conformal invariant. The presence of the diagonal term in the two-point function leads to the conclusion that in the quantum theory of the Thirring model a spontaneous breakdown of a symmetry should take place. In order to verify the above-mentioned consideration, let us perform the γ_5-gauge transformation with the generator being the pseudo-charge $\tilde{Q}$. (Consider for simplicity the case $\alpha_1=\alpha_2$ and $\beta_1=\beta_2$.)

$$\psi(x)\to \mathrm{e}^{\mathrm{i}\lambda\gamma_5}\psi(x), \qquad \phi(x)\to\phi(x), \qquad \tilde{\phi}(x)\to\tilde{\phi}(x), \tag{3.10}$$

where λ is a numerical parameter, and $\gamma_5=\gamma_0\gamma_1$.

Obviously, the equation of the Thirring model

$$\mathrm{i}\gamma^\mu\partial_\mu\psi(x)=-g{:}J_\mu(x)\gamma^\mu\psi(x){:} \tag{3.11}$$

is invariant with respect to this transformation. If the vacuum state $|0\rangle$ was supposed to be invariant too, then the function $\Delta_{ij}(x)$ would satisfy the equation

$$\Delta_{ij}(x)=[\mathrm{e}^{\mathrm{i}\lambda\gamma_5}\Delta(x)\,\mathrm{e}^{\mathrm{i}\lambda\gamma_5}]_{ij} \tag{3.12}$$

and in particular for the diagonal terms (taking into account that γ_5 is also diagonal):

$$\Delta_{kk}(x)=\exp[2\mathrm{i}\lambda(-1)^k]\Delta_{kk}(x).$$

Hence it follows that $\Delta_{kk}(x)=0$, which is a contradiction to equation (3.8). Therefore the vacuum cannot be invariant with respect to the symmetry (3.10), which means that in the quantum Thirring model the γ_5-symmetry is spontaneously broken.

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Appendix 1.

The following commutation functions have been used throughout the paper:

$$D(x) = -\tfrac{1}{2}\epsilon(x^0)\theta(x^2), \qquad D^+(x)+D^-(x) = \mathrm{i}D(x),$$
$$D^\pm(x) = \mp(1/4\pi)\ln(-\mu^2x^2 \pm \mathrm{i}0x^0),$$
$$\tilde{D}(x) = -\tfrac{1}{2}\epsilon(x^1)\theta(-x^2), \qquad \tilde{D}^+(x)+\tilde{D}^-(x) = \mathrm{i}\tilde{D}(x),$$
$$\tilde{D}^\pm(x) = \pm\frac{1}{4\pi}\ln\frac{x^0-x^1\mp\mathrm{i}0}{x^0+x^1\mp\mathrm{i}0}.$$

Appendix 2.

We write down the commutation relations of the 'spinor' Thirring field $\psi(x)$ with the generators of the conformal group representations.

$$[P_\mu, \psi(x)] = \mathrm{i}\partial_\mu\psi(x), \tag{A2.1}$$

$$[M_{\mu\nu}, \psi(x)] = -\mathrm{i}(x_\mu\partial_\nu - x_\nu\partial_\mu)\psi(x) + \epsilon_{\mu\nu}:(\alpha S + \beta L\gamma_5)\psi(x):, \tag{A2.2}$$

$$[D, \psi(x)] = \mathrm{i}x^\mu\partial_\mu\psi(x) + :(\alpha L + \beta S\gamma_5)\psi(x):, \tag{A2.3}$$

$$[K_\mu, \psi(x)] = -\mathrm{i}(2x_\mu x_\nu - g_{\mu\nu}x^2)\partial^\nu\psi(x) + 2x_\mu:(\alpha L + \beta S\gamma_5)\psi(x): + 2\epsilon_{\mu\nu}x^\nu:(\alpha S + \beta L\gamma_5)\psi(x):, \tag{A2.4}$$

where α and β are arbitrary parameters, taking continuous values and

$$S = (1/2\sqrt{2\pi})[b^+(0) + b^-(0)], \qquad L = (1/2\sqrt{2\pi})[a^+(0) + a^-(0)].$$

The constant operators $a^\pm(0)$ and $b^\pm(0)$ are defined by Hadjiivanov *et al* (1979) and their commutation relations with the field $\psi(x)$ are

$$[a^\pm(0), \psi(x)] = \mp(\mathrm{i}\alpha/\sqrt{2\pi})\psi(x), \qquad [b^\pm(0), \psi(x)] = \mp(\mathrm{i}\beta/\sqrt{2\pi})\gamma_5\psi(x).$$

The two conserved charges have also been found there in terms of these operators:

$$Q \equiv \int_{-\infty}^{\infty} \mathrm{d}x^1\, \partial_0\phi(x) = \tfrac{1}{2}\mathrm{i}\sqrt{\pi}[a^-(0) - a^+(0)],$$

$$\tilde{Q} \equiv \int_{-\infty}^{\infty} \mathrm{d}x^1\, \partial_0\tilde{\phi}(x) = -\tfrac{1}{2}\mathrm{i}\sqrt{\pi}[b^-(0) - b^+(0)].$$

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